Logarithmic inequalities

REMEMBER: For $\log_a b = x \Leftrightarrow b = a^x$ **,** $(a>0, a\neq 0)$ **is**

1) When is the basis greater than 1 ($a>1$) sign of inequality remains the same

2) When is the basis between 0 and 1 ($0<a<1$), rotate the sign of inequality

1) Solve the inequalities:

a) $\log_2(3x+4)\geq 0$
b) $\log_{\frac{1}{2}}(4x-3)<0$
c) $\log_2(3x-5)<1$

Solution:

a)

$\log_2(3x+4)\geq 0$ condition: $3x+4>0$

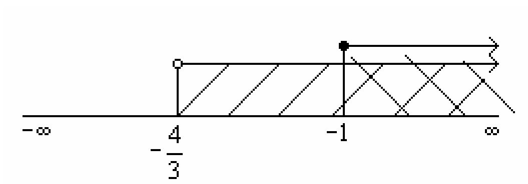
$3x+4\geq 2^o$ $\quad 3x>-4$

$3x+4\geq 1$ $\quad x>-\frac{4}{3}$

$3x\geq -3$

$x\geq -1$

Now, observe solution and the condition:



Finally: $x\in[-1,\infty)$

b)

$\log_{\frac{1}{2}}(4x-3)<0$ condition: $4x-3>0$

$4x>3$

$x>\frac{3}{4}$

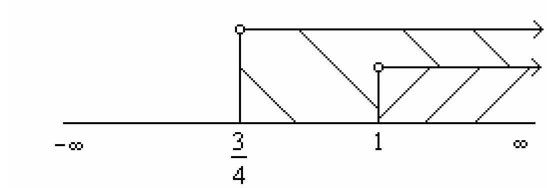
$4x-3>\left(\frac{1}{2}\right)^o$

$4x-3>1$

$4x>4$

$x>1$

observe solution and the condition :



Solution: $x\in(1,\infty)$

c)

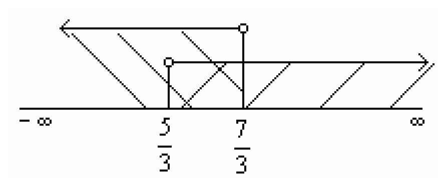
$\log_2(3x-5) < 1$ condition: $3x-5>0$

$3x-5 < 2^1$ $3x>5$

$3x-5<2$ $x>\frac{5}{3}$

$3x<7$

$x<\frac{7}{3}$



$x \in \left(\frac{5}{3}, \frac{7}{3}\right)$

2) Solve the inequalities:

a) $\log(x-2) > \log x$
b) $\log_{0,5}(2x+6) > \log_{0,5}(x+8)$

Solution:

a)

$\log(x-2) > \log x$ condition : $x-2>0$ and $x>0$

$x-2>x$ $x>2$ and $x>0$

$x-x>2$ So: $x>2$

$0x>2$

This has no solutions, and the whole inequalities has no solutions!

b)

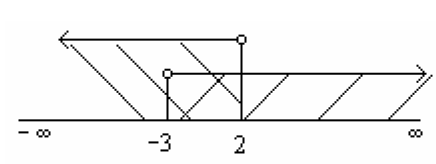
$\log_{0,5}(2x+6) > \log_{0,5}(x+8)$

$2x+6<x+8$ $2x+6>0 \wedge x+8>0$

$2x-x<8-6$ $x>-3 \wedge x>-4$ ⟶ $x>-3$

$x<2$

together:



$x \in (-3,2)$ final solution

3) Solve the inequalities:

a) $\log_3(x^2-5x+6)<0$

b) $\log_{0,5}(x^2-4x+3)\geq -3$

Solution:

a)

$$\log_3(x^2-5x+6)<0$$

$$x^2-5x+6<3^o$$

$$x^2-5x+6<1$$

$$x^2-5x+5<0$$

$$x_{1,2}=\frac{5\pm\sqrt{5}}{2}$$

$$x_1=\frac{5+\sqrt{5}}{2}\approx 3{,}62$$

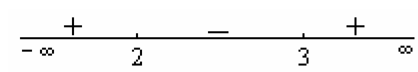
$$x_2=\frac{5-\sqrt{5}}{2}\approx 1{,}38$$

$$x^2-5x+6>0$$

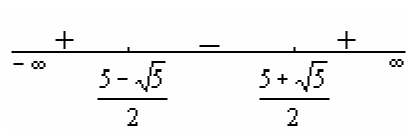
$$x_{1,2}=\frac{5\pm 1}{2}$$

$$x_1=3$$

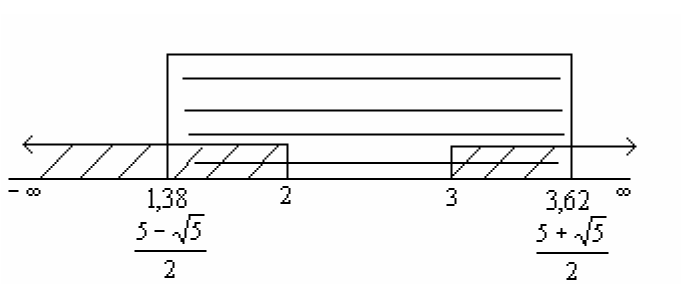
$$x_2=2$$



$$x\in(-\infty,2)\cup(3,\infty)$$



$$\longrightarrow \quad x\in\left(\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right)$$



$$x\in\left(\frac{5-\sqrt{5}}{2},2\right)\cup\left(3,\frac{5+\sqrt{5}}{2}\right)$$ Is final solution

b)

$$\log_{0,5}(x^2-4x+3)\ge -3$$

$$x^2-4x+3\le (0,5)^2$$

$$x^2-4x+3\le \left(\frac{1}{2}\right)^{-3}$$

$$x^2-4x+3\le 2^3$$

$$x^2-4x+3\le 8$$

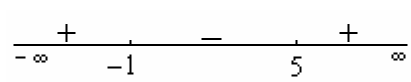
$$x^2-4x+3-8\le 0$$

$$x^2-4x-5\le 0$$

$$x_{1,2}=\frac{4\pm 6}{2}$$

$$x_1=5$$

$$x_2=-1$$



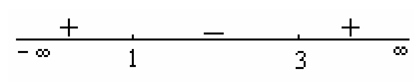
$$x\in[-1,5]$$

condition: $x^2-4x+3>0$

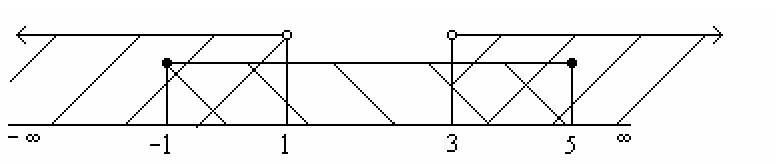
$$x_{1,2}=\frac{4\pm 2}{2}$$

$$x_1=3$$

$$x_2=1$$



together:



Finally: $x\in[-1,1)\cup(3,5]$